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Temperature stresses in Functionally graded (FGM) material plates using deformation theory – Analytical approach

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Highlights

- First Order Shear Deformation Theory.
- Simply Supported FGM plates subjected to Thermal Loads.
- Analytical Formulations and solutions using Navier's Solution Technique.
- Benchmark results for different aspect ratio, side-to-thickness ratio for different materials.

Abstract

The advancement in the field of composites lead to the development of a new material called Functionally graded materials (FGMs). The concept of grading two distinct materials is introduced in FGMs to withstand high temperature variations across thin section. In this study analytical formulations and theoretical solutions were developed for the FGM plates subjected to thermal loads using refined computational model based on First Order Shear Deformation Theory (FSDT). Material properties are based on Power-law function. The effects of thermal stresses are studied for constant, linear and nonlinear variation of temperatures across the thickness of the plate, whereas in-plane is assumed to be sinusoidal. The accuracy of solutions are established with the literature. Parametric studies are performed and numerical results are presented for the simply supported FGM plates. Comparative studies are performed between various temperature profiles and bench mark results are presented for displacements, in-plane and transverse stresses.

Introduction

Functionally graded materials (FGMs) are the advanced materials in the field of composites, which are extensively used in aerospace, nuclear, automotive fields etc., involving high temperature fluctuations. In this material, the concept of layers in composite laminated plates is replaced by adopting continuous gradation of materials. Due to this property of FGM, the effect of debonding or delamination failures in laminates are suppressed. This idea of gradually changing the properties of the materials was first proposed by Shen and Bever [1] for composites and polymeric materials in 1972. Most of these materials were used as coating materials to resist thermal stresses. The first application was carried out at National Aerospace Laboratories of Japan in 1984, to create fuselage and nosecone parts of a space vehicle which can withstand a temperature gradient of 1600K across a 10mm thick section [2]. These FGMs inherit the physical and chemical properties of both the materials and found to exhibit high bond strength with excellent insulation properties, thermal resistivity, high strength and stiffness.

Most of the theories that are proposed for the analytical evaluation of FGM plates are extended from laminate plate theories. The approximations in non-symmetric plates and its effects on bending-stretching coupling terms are discussed by Mian and Spencer [3]. Exact solutions are the most accurate methods for predicting the plate responses, but due to mathematical complexities involved, it has to be reduced to two dimensional forms. Reddy and Cheng [4] and Wang et al. [5] predicted thermo-elastic responses in FGM plates using asymptotic methods. Yihunie M B et al. [6] discussed the effect of volume fraction index on thermal properties like conductivity and expansion coefficient in radial directions in a thick walled FGM cylinders. It was observed that the temperature distribution is not homogeneous. Anthony Xavior et al., [7] manufactured a synthesized FGM by varying its densities along width using additive manufacturing technology. Chung and Chang [8] investigated the elastic behaviour of rectangular FGM plates subjected to constant temperature throughout the plate, using Power law (P-FGM), Sigmoidal (S-FGM) and exponential (E-FGM) variations. Nosier and Fallah [9] used First Order Shear Deformation Theory (FSDT) to investigated the responses in circular FGM plates subjected to mechanical and thermal loads using power law variation of material properties. Temperature is assumed to vary according to 1-D steady state heat equation. It was observed that the material properties and temperature profiles had a significant effect on the stress. In the present study, analytical evaluation of thermal stresses in a simply supported rectangular FGM plate subjected to constant, linear and nonlinear variation of thermal loads is carried out. Comparative studies were performed on various material properties, temperature profiles and plate parameters.

Section snippets

Theoretical Formulations:

The First Order Shear Deformation Theory (FSDT) is based on classical plate theory and was first proposed by Whitney and Pagano [10] in 1970, the displacement model at any point in an FGM plate is expressed as given in Eq. (1),

$$u\left(x,y,z\right)=u_0\left(x,y\right)+z\theta_x\left(x,y\right),$$
$$v\left(x,y,z\right)=v_0\left(x,y\right)+z\theta_y\left(x,y\right),$$
$$w\left(x,y,z\right)=w_0\left(x,y\right).$$

The terms u, v and w are the displacements of a general point (x, y, z) in×,y and z directions respectively. The parameters u_0, and v_0 are the in-plane displacements and w_0 is the transverse…

Equilibrium equations

The governing equations of equilibrium are obtained using Principle of Minimum Potential Energy (PMPE). The corresponding equations related to first order computational model are,$(1)\delta \mathrm{u}_0:\left(\frac{\partial \mathrm{N_x}}{\partial \mathrm{x}}+\frac{\partial \mathrm{N_{xy}}}{\partial \mathrm{y}}\right)=0(2)\delta \mathrm{v}_0:\left(\frac{\partial \mathrm{N_y}}{\partial \mathrm{y}}+\frac{\partial \mathrm{N_{xy}}}{\partial \mathrm{x}}\right)=0$ $(3)\delta \mathrm{w}_0:\left(\frac{\partial \mathrm{Q_x}}{\partial \mathrm{x}}+\frac{\partial \mathrm{Q_y}}{\partial \mathrm{y}}+\mathrm{p_z^+}\right)=0(4)\delta\theta_\mathrm{x}:\left(\frac{\partial \mathrm{M_x}}{\partial \mathrm{x}}+\frac{\partial \mathrm{M_{xy}}}{\partial \mathrm{y}}-\mathrm{Q_x}\right)=0(5)\delta\theta_\mathrm{y}:\left(\frac{\partial \mathrm{M_y}}{\partial \mathrm{y}}+\frac{\partial \mathrm{M_{xy}}}{\partial \mathrm{x}}-\mathrm{Q_y}\right)=0$Here ($N_x$, N_y, N_{xy}), (M_x, M_y, M_{xy}) and (Q_x, Q_y) respectively denotes in-plane, bending and shear stress resultants due to thermal loads, which are expressed in Eq. (9)$\left\{\begin{matrix}Q_x\\Q_x^*\end{matrix}\right\}=[D]\left\{\begin{matrix}\theta_x\\\frac{\partial w_0}{\partial x}\end{matrix}\right\}+[D']\left\{\begin{matrix}\theta_y\\\frac{\partial w_0}{\partial y}\end{matrix}\right\},\left\{\begin{matrix}Q_y\\Q_y^*\end{matrix}\right\}=[E]\left\{\begin{matrix}\theta_y\\\frac{\partial w_0}{\partial y}\end{matrix}\right\}+[E']\left\{\begin{matrix}\theta_x\\\frac{\partial w_0}{\partial x}\end{matrix}\right\},\{N\}$…

Analytical solutions

In order to solve the boundary value problem for thermal stresses in a rectangular simply supported FGM plate, Navier's solution technique using the double Fourier series is adopted. The solution for the Fourier amplitudes is obtained using Eq. (11),$[X]_{5\times5}\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_x \\ \theta_y \end{Bmatrix}_{5\times1} = \begin{Bmatrix} 0 \\ 0 \\ P_z^+ \\ 0 \\ 0 \end{Bmatrix}_{5\times1} + \{F_T\}_{5X1}$

For any fixed values of m and n. The elements of the coefficient matrix [X] is given by Eq. (12),$X_{1,1} = A_{1,1}\alpha^2 + B_{1,1}\beta^2$ $X_{1,2} = A_{1,2}\alpha\beta + B_{1,2}\alpha\beta X_{1,3} = 0 X_{1,4} = A_{1,3}\alpha^2 + B_{1,3}\beta^2 X_{1,5} = A_{1,4}\alpha\beta + B_{1,4}\alpha\beta X_{2,2} = A_{2,2}\beta^2 + B_{1,2}\alpha^2$ $X_{2,3} = 0 X_{2,4} = A_{2,3}\alpha\beta + B_{1,3}\ \alpha$...

Numerical results and discussion

In this section, the numerical examples solved are described and discussed for establishing the accuracy of the solutions obtained using FSDT model. A shear correction factor of 5/6 is used in obtaining the results. For all problems described and discussed below, a simply supported rectangular FGM plate with SS-1 boundary conditions is considered. Closed form solutions obtained using *Navier's* solution technique for the above geometry and loading are presented. The material sets used for the…

Conclusion

Analytical formulations and theoretical solutions are presented for thermal displacements and stresses in a simply supported FGM plate subjected to constant, linear and nonlinear variation of thermal loads using refined computational model based on First Order Shear Deformation Theory (FSDT). From the present investigation it is observed that the effect of assumed temperature profile plays a crucial role in predicting the thermal plate responses. The temperature field based on steady state heat …

CRediT authorship contribution statement

D.M. Sangeetha: Conceptualization, Data curation, Investigation, Methodology, Writing - original draft. **D.T. Naveenkumar:** Formal analysis, Methodology, Software. **V. Vınaykumar:** Writing - review & editing. **KE. Prakash:** Formal analysis, Supervision.…

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.…

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